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Analysis Error Student In Learning Geometry As Base Understanding Non-Euclidean Geometry : An Overview Literature Based on Newman's Error Analysis

Dosta Ernauli Siregar[1], Lasvina Br. Purba[2], Jesica Dhea Chyntia[3], KMS Muhammad Amin Fauzi[4]

[1234] Universitas Negeri Medan, Indonesia
* dosta123siregar@gmail.com

ABSTRACT

This study aims to identify the dominant error patterns of students in geometry learning based on Newman's Error Analysis (NEA) and to examine their implications for readiness to understand Non-Euclidean Geometry. The method employed is a Systematic Literature Review (SLR) of 10 empirical articles published between 2019 and 2026. Data were extracted based on the five stages of NEA — reading, comprehension, transformation, process skill, and encoding — and subsequently analyzed through narrative synthesis, percentage comparison, and descriptive effect size. The results indicate that comprehension errors and encoding errors are the most dominant categories, with the highest percentages of 56.92% and 50%, respectively, followed by transformation errors, which consistently fall within a moderate effect range (20–40%), while reading errors and process skill errors are classified as low. The primary causes of errors include weak understanding of geometric concepts, inability in spatial visualization, limited mathematical language, and lack of procedural precision. The findings contribute as a diagnostic foundation for designing more effective geometry learning, while simultaneously serving as a conceptual bridge toward higher-level geometry.

Keywords: Error Analysis, Geometry, Newman's Error Analysis, Non-Euclidean Geometry

INTRODUCTION

Geometry is one of the most strategic and challenging branches of mathematics for students. As a field of science directly related to the understanding of space, shape, and spatial relationships, geometry provides a conceptual foundation for many disciplines, including physics, architecture, and engineering (Wahyuni, Efuansyah, & Fitriyana, 2026). However, empirical data consistently shows that Indonesian students face serious difficulties in this material. The 2022 PISA results ranked Indonesia 70th out of 81 countries with an average score of 366, indicating weak mathematical literacy covering the domains of geometry and measurement (OECD, 2023). TIMSS data also consistently ranks Indonesia at the bottom in mathematics achievement, including geometry (Safitri, Sugiarti, & Hutama, 2019).

Student errors in geometry are not random events, but rather have patterns that can be identified and analyzed systematically. *Newman's Error Analysis* (NEA), developed by Anne Newman (1977), maps the cognitive process of solving math problems through five hierarchical stages: reading, comprehension, transformation, process skills, and encoding. NEA not only identifies the location of errors but also reveals why they occur (Atika, Yuhana, & Syamsuri, 2023) , making it an invaluable diagnostic tool for teachers. (Azkiana, Nopriana, & Aminah, 2025) .

Although a number of studies have applied NEA to specific mathematical topics, there are two significant gaps that have not been filled. (Hamid, Suryani, & Yusri, 2023). First, there has not been a study that comprehensively synthesizes student error patterns in geometry learning from various contexts and levels of education. (Fadilah & Bernard, 2021) . Second, and more fundamentally, almost no research explicitly links Euclidean geometry error patterns with their implications for readiness to learn Non-Euclidean Geometry—a geometric system that goes beyond Euclid's axioms and is increasingly relevant in higher mathematics and modern physics (Azkiana et al., 2025). This research fills both gaps through a *Systematic Literature Review* (SLR) approach to 10 articles (2019–2026), with the objectives of: (1) identifying dominant patterns and types of students' errors in geometry based on NEA; (2) analyzing the causal factors; and (3) examining the implications of the findings for learning Non-Euclidean Geometry as a contribution *to improving novelty.* (Hendrayanto, Wijayanto, Wahmad, & Widodo, 2021).

LIBRARY REVIEW

Newman's Error Analysis: A Conceptual Framework

Newman (1977) developed NEA based on the premise that solving mathematical problems is a series of hierarchical cognitive stages, where failure at one stage automatically hinders the next stage (Hamid et al., 2023) . The five stages are: (1) reading error, students fail to read symbols or keywords in the problem (Hamid, Suryani, & Yusri, 2023); (2) *comprehension error,* students read the problem correctly but fail to understand the overall meaning of the problem so they cannot identify what is known and what is asked (Wahyuni et al., 2026); (3) *transformation error,* students understand the problem but fail to transform it into an appropriate mathematical model, including choosing the wrong formula or method (Kristiyaningrum & Sumarni, 2023); (4) *process skill error,* students choose the correct procedure but make errors in the computational or algorithmic steps (Hendrayanto et al., 2021) ; and (5) *encoding error,* students are unable to present the final answer correctly in the context of the problem, for example not writing the units or not making a conclusion (Kitchenham, 2004). This hierarchical nature of NEA is what makes it a *powerful* diagnostic tool : it not only shows the product of the error, but reveals its cognitive roots. (Yuniar, Lidinillah, & Apriani, 2025) .

Difficulties in Learning Geometry

The difficulty of learning geometry rests on three mutually reinforcing dimensions. First, conceptual abstraction: geometric objects such as points, lines, and planes are abstract and must be constructed through mental processes that require cognitive maturity (Kristiyaningrum & Sumarni, 2023) . Second, spatial visualization: problems with geometric shapes require the ability to mentally manipulate three-dimensional objects, and many students fail at this stage, particularly in understanding components such as painter's lines or curved surfaces (Suryani, 2024; Azkiana et al., 2025). Third, mathematical language: geometry uses precise terminology that differs from everyday use, so weaknesses in reading symbols or interpreting keywords become the gateway to a chain of errors that affect the entire problem-solving process (Newman, 1977) .

Non-Euclidean Geometry and Its Relevance

Euclid's geometry, which applies to the plane and is based on five postulates — including the parallel postulate that through a point outside a line there is only one parallel line — has dominated the study of mathematics for more than two millennia. (Suryani, 2024). In the 19th century, Lobachevsky, Bolyai, and Riemann proved that a consistent geometric system could be constructed by modifying the parallel postulate (Ozerem, 2012) . Hyperbolic geometry (Lobachevsky-Bolyai) assumes that more than one parallel line can be drawn through a point, so that the sum of the angles of a triangle is always less than 180° (Kristiyaningrum & Sumarni, 2023) . In contrast, Elliptic (Riemann) Geometry assumes no parallel lines, with the sum of the angles of a triangle always greater than 180°—relevant in the context of a spherical surface. Riemannian geometry even serves as the mathematical foundation of Einstein's General Theory of Relativity. The cognitive leap required to understand these systems is enormous: students must be able to shed years of Euclidean geometric intuition and construct a new understanding based on different axioms. The quality of a student's foundation in Euclidean geometry is a key determinant of their ability to make this leap. (White, 2009) .

Synthesis of Error Patterns from the Literature

A cross-study analysis of 10 articles revealed a consistent pattern: *comprehension errors* and *encoding errors* dominated across contexts and educational levels. Wahyuni et al. (2026) reported the highest *comprehension errors* (56.92%) in quadrilaterals and plane-sided solids, while Fadilah and Bernard (2021) found a 51.4% prevalence *of comprehension errors* in the context of congruence and similarity. Yuniar et al. (2025) confirmed that this pattern is present from elementary school, with a 35.79% prevalence *of comprehension errors* in geometric numeracy problems. On the encoding side, Fadilah and Bernard (2021) reported the highest rate (50%), followed by Hamid et al. (2023), who listed *encoding errors* as the largest category in their study. *Transformation errors* consistently ranked second or third, ranging from 13.3 to 32.1%, reflecting a systemic barrier in the connection between conceptual understanding and formal mathematical representation. (Wahyuni et al., 2026) . In contrast, *reading errors* are the lowest category and are not even found at all in some studies (Safitri et al., 2019) .

METHOD

This study employed *a Systematic Literature Review* (SLR) (Kitchenham, 2004), a structured secondary research method designed to synthesize evidence from primary studies in an objective and replicable manner. SLR was chosen because it can produce a comprehensive picture of the research topic while minimizing the selection bias common in conventional literature reviews. (Suryani, 2024). The inclusion criteria included: (1) empirical research reports analyzing student errors in geometry learning; (2) using NEA as the main analytical framework; (3) published in indexed scientific journals in 2019–2026; (4) available in Indonesian or English; and (5) presenting quantitative data (percentage or frequency) per NEA category that allows for analytical comparison. Articles that only discussed error analysis in general without referring to NEA, or those with insufficient data for meaningful extraction, were excluded. (Suryani, 2024) .

The search was conducted systematically through Google Scholar, SINTA, and accredited national journal repositories using the keywords: 'geometric error analysis', *'Newman's Error Analysis',* 'mathematics student errors', and their English variations. Articles were screened in stages based on the relevance of the title and abstract, then through full-text reading until 10 articles were obtained that met all inclusion criteria. (Singh, Rahman, & Hoon, 2010). Data were extracted in a structured manner including article identity, research characteristics (level of education, geometry topic, number of subjects), percentage or frequency of errors per NEA category, dominant categories, and reported causal factors. (Wahyuni et al., 2026) . Data analysis was conducted through three complementary approaches: (1) narrative synthesis, which combines findings descriptively and analytically to identify patterns and divergences across studies; (2) percentage comparison by identifying dominant error hierarchies across studies; and (3) descriptive *effect size* with a percentage >40% categorized as a large effect (high significance), 20–40% as a medium effect, and <20% as a small effect

RESULTS AND DISCUSSION

Article Overview

The ten articles analyzed covered the publication period 2019–2026, covering a variety of geometry topics, educational levels, and research locations. Table 1 presents a summary of the error percentage data per NEA category from all articles.

Table 1. Synthesis of Student Errors Based on NEA from 10 Articles

Note: R = Reading; C = Comprehension; T = Transformation; PS = Process Skills; E = Encoding; Dom. = Dominant

No	Article (Author, Year)	Geometry Topics	R (%)	C (%)	T (%)	PS (%)	E (%)	Dominant
1	Hendrayanto et al. (2021)	3D Geometry (Spatial Figures)	0	There is	Dominant	There is	There is	T, C, PS, E

No	Article (Author, Year)	Geometry Topics	R (%)	C (%)	T (%)	PS (%)	E (%)	Dominant
2	Kristiyaningrum & Sumarni (2023)	Geometric Transformation	10.0	13.3	13.3	31.7	13.3	PS
3	Hamid, Suryani & Yusri (2023)	Flat-Sided Space Structure (SMP)	7.2	15.2	22.4	19.2	34.4	E, T
4	Suryani (2024)	Geometry & Measurement (AKM)	0	Dom.	There is	There is	Dom.	C, E
5	Azkiana, Nopriana & Aminah (2025)	Surface Area of a Sphere	There is	There is	There is	There is	Dom.	E, PS
6	Yuniar, Lidinillah & Apriani (2025)	Geometry & Measurement Numeration (SD)	0	35.79	17.89	16.84	4.21	C
7	Wahyuni, Efuansyah & Fitriyana (2026)	Quadrilaterals & Flat-Sided Space Structures	0	56.92	23.08	12.31	7.69	C
8	Safitri, Sugiarti & Hutama (2019)	Flat Shapes (SD)	13.3	10.89	27.62	15.73	32.45	E, T
9	Fadilah & Bernard (2021)	Congruence & Similarity	23.6	51.4	32.1	13.6	50.0	C, E
10	Atika, Yuhana & Syamsuri (2023)	SPLDV (Geometry Context)	There is	Dom.	There is	There is	Dom.	C, E

Table 1 shows that *comprehension, transformation,* and *encoding* were the categories that appeared most consistently across the studies. Reading error was the only category that was absent or scored zero in five of the ten studies, providing a strong initial signal about where students' primary cognitive barriers lie. (Singh et al., 2010) .

Dominant Errors and Their Analysis

Comprehension errors are the most dominant category across studies, reaching a large effect in Wahyuni et al.'s (2026) study with 56.92% and Fadilah & Bernard's (2021) study with 51.4%. Yuniar et al.'s (2025) confirmed that this problem is present since elementary school (35.79%). This dominance is diagnostic: failure at the *comprehension stage* creates a domino effect that automatically damages *the transformation, process skills,* and *encoding stages,* because each stage is built on a foundation of understanding that has already failed. Wahyuni et al.'s (2026) explain that the main obstacle lies in weak representational and metacognitive abilities, not in the technical ability to read text; students memorize definitions without understanding their conceptual meaning. (Wahyuni et al., 2026) .

Encoding errors were the second most common category, with a large effect in Fadilah & Bernard's (2021) study of 50%, and close to the large effect in Safitri et al.'s (2019) study of 32.45%. Hamid et al. (2023)

recorded *encoding* as the most common type of error (43 out of 123 total errors). *Encoding errors* have two distinct sources: as a result of failure in previous stages, and as independent failures where students have performed the computation correctly but failed to write down the units or conclusions (Usiskin, 2018) . This second source reflects a lack of mathematical interpretation skills and the connection between mathematical formalism and real-world contexts (Youngu et al., 2022).

Transformation errors consistently have a moderate effect (20–40%) with a range of 13.3–32.1% across most studies (Safitri et al. (2019): 27.62%; Fadilah & Bernard (2021): 32.1%; Wahyuni et al. (2026): 23.08%). This consistency suggests that *transformation errors* are not a situational issue, but rather a structural challenge in geometry learning: students are able to understand problems verbally but fail to translate them into formal, operational mathematical representations. *Reading errors* and *process skill errors* consistently have small effects. (Yuniar et al., 2025) . No *reading errors were found* in five studies (White, 2009) indicates that students' basic text *decoding abilities* are not the main obstacle — the obstacle lies in deeper cognitive processes. (Safitri et al., 2019) .

Factors Causing Errors

Four dominant factors were identified across studies. First, conceptual understanding weaknesses: Hendrayanto et al. (2021) concluded that the primary cause of errors was a lack of understanding of geometric concepts, while Kristiyaningrum and Sumarni (2023) found hidden misconceptions that only came to light when students were confronted with in-depth application problems. Second, spatial visualization inadequacies: Suryani (2024) identified the inability to visualize three-dimensional shapes as a significant source of *comprehension errors,* and Azkiana et al. (2025) found difficulty visualizing the curved surface of a sphere as a cause of *transformation errors.* Third, limitations in mathematical language: Safitri et al. (2019) found that a lack of mastery of symbols and problem instructions contributed to various types of errors, while Fadilah and Bernard (2021) noted that students misidentified keywords that define mathematical operations. Fourth, procedural inaccuracy: Kristiyaningrum and Sumarni (2023) found that *process skill errors* (31.7%) were largely due to segment displacement errors, and Azkiana et al. (2025) noted that students performed calculations correctly but forgot to write down the units. Suryani (2024) linked the habit of using multiple-choice questions as a factor *in encoding errors,* where students were not accustomed to connecting mathematical answers to the context of the questions. (Yuniar et al., 2025) .

Effect Size Deskriptif

Based on the established categorizations, *comprehension errors* achieved a large effect (>40%) in two studies (Wahyuni et al., 2026: 56.92%; Fadilah & Bernard, 2021: 51.4%) and a moderate effect in most of the other studies—making it the category with the highest significance across studies. *Encoding errors* achieved a large effect in one study (Fadilah & Bernard, 2021: 50%) and a moderate effect in three others. *Transformation errors* consistently had a moderate effect (20–40%) in seven of the ten studies, indicating structural significance that requires serious attention. *Reading errors* and *process skill errors* predominantly had small effects (<20%). This distribution provides clear guidance for intervention priorities: strengthening conceptual understanding and mathematical interpretation skills should be the top priority, followed by improving transformation skills across representations. (Atika et al., 2023) .

Implications for Learning Non-Euclidean Geometry

The above findings have direct and profound implications for the prospects for learning Non-Euclidean Geometry. The central argument of this study is that the dominant errors in Euclidean geometry are not simply technical obstacles that can be overcome with more practice, but rather reflect a fundamental cognitive deficit that will be compounded when students are confronted with the demands of abstraction in Non-Euclidean Geometry. (Kristiyaningrum & Sumarni, 2023) .

comprehension errors in Euclidean geometry, even the most intuitive geometric system, mean that students are not yet able to construct accurate mental representations of geometric situations. (Hendrayanto et al., 2021) . Understanding that the sum of the angles of a hyperbolic triangle is always less than 180° requires students to first understand in depth why the sum of the angles of a Euclidean triangle is exactly 180° and the implications of the parallel postulate. Without a strong foundation *of comprehension, understanding of axiom* *modification in non-Euclidean systems will be very limited.* Consistent *transformation errors indicate that the connection between visual-verbal representations and formal mathematical representations has not been well established.* (Azkiana et al., 2025) . This ability becomes even more critical in Non-Euclidean Geometry, where everyday visual intuition can be misleading—for example, in hyperbolic geometry, the concept of a 'straight line' must be visualized through special models such as the counter-intuitive Poincaré disk. High *encoding errors* reflect an inability to interpret mathematical results within the prevailing geometric framework—an ability that becomes even more critical in Non-Euclidean Geometry, where the meaning of 'distance' and 'angle' depends on the axioms of the system used. (Hamid et al., 2023) .

Overall, these findings confirm that meaningful learning of Non-Euclidean Geometry is unlikely to succeed without first ensuring a solid conceptual understanding of Euclidean geometry—not just mastery of procedures, but rather a deep understanding of why certain geometric properties hold, how relationships between concepts are formed, and what the geometric meaning of a mathematical result is. Addressing *comprehension errors, transformation errors,* and *encoding errors* in Euclidean geometry is an indispensable prerequisite

CONCLUSION

This SLR study synthesizes student error patterns in geometry learning based on NEA from 10 articles (2019–2026). Three main findings can be formulated. First, *comprehension errors* are the most dominant error across studies (large effect up to 56.92%), followed by *encoding errors* (large effect up to 50%) and transformation errors which consistently have a moderate effect (13.3–32.1%). *Reading errors* and *process skill errors* show small effects. Second, four main causal factors identified are: weak understanding of geometric concepts, inability to visualize spatially, limited mastery of mathematical language, and lack of procedural accuracy. Third, as a contribution *to improvement novelty,* the dominant error patterns in Euclidean geometry, especially *comprehension, transformation,* and *encoding errors,* are fundamental obstacles that will multiply the difficulty of learning Non-Euclidean Geometry, which demands abstraction and connections across representations at a much higher level.

Based on these findings, teachers are advised to use NEA as an active diagnostic tool before designing interventions, and orient geometry learning toward strengthening conceptual understanding through problem-based approaches and visualization. Before introducing Non-Euclidean Geometry, educators need to verify that students have a well-developed foundation of spatial visualization and connections across representations. Further research that develops specific interventions to address *comprehension* and *transformation errors,* as well as empirically examines the relationship between mastery of Euclidean geometry and readiness to learn Non-Euclidean Geometry, is urgently needed.

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